condition there is for every set of angles A, B, C one ratio $a : b$ for which the locus is a straight line.

The tangent to the curve at O is parallel to the line S_0R_0 and at any point H it is parallel to the line SR. The curve is concave downward

$$\text{if } \frac{a}{b} < \frac{1 + \sqrt{1 + PQ}}{P}, \qquad \text{upward if } \frac{a}{b} > \frac{1 + \sqrt{1 + PQ}}{P}.$$

The special case $B = C = 0$ is of interest. The equation now becomes $\cos Ay^2 + 2xy - 2bx + 2ay = 0$. Using rectangular coördinates, u and v, with the old origin and X-axis,

$$u = x + y \cos A, \qquad v = y \sin A,$$

the equation becomes

$$\cot A \cdot v^2 - 2uv + 2ku - 2hv = 0,$$

where $h = a + b \cos A$, $k = b \sin A$. The center is at the point $(-a, k)$ which is the point where S_0S meets the now horizontal line T_0R_0. One of the asymptotes is the line R_0T_0. The curve is concave downward in this case and the uneroded portion tends to become horizontal as erosion progresses. It might be expected that R_0T_0 is always an asymptote, but this is the case only when $\angle B = \not\angle C$, *i. e.*, $P = 0$.

THE OBSOLETE IN MATHEMATICS.

By G. A. MILLER, University of Illinois.

The present article was suggested by extravagant claims made in the advertisements of some general works of reference. As an extreme case we may refer to recent advertisements of the *Mathematical Dictionary and Cyclopedia of Mathematical Science* by Charles Davies and W. G. Peck, which contained the following statements: "A standard work for 60 years." "Definitions of all terms employed in mathematics, an analysis of each branch and the whole as forming a single science."

The claims represented by these quotations, which many readers of this MONTHLY have doubtless seen, present a serious affront to our intelligence, since any mathematical reader should at once recognize that these claims must be false even if he has never heard of the work in question. Hence it is not the object of the present article to establish their extravagance. Those who have only a slight knowledge of the mathematical advances during the last sixty years know that a work written at the beginning of this period could not contain definitions of all the important terms now employed in mathematics.

On the other hand, it will probably be of interest to those readers who have

not had an opportunity to examine the mathematical dictionary mentioned above to be furnished a few evidences from its pages of the great mathematical advances made during the last sixty years, especially in our own country. It may be desirable to emphasize the last phrase in the preceding sentence since American mathematics was not then in as close contact with the best mathematics of the world as it is to-day.

As evidence of this fact we may note that the dictionary under consideration does not contain the term *determinants,* although the subject of determinants had been extensively developed in Europe at a much earlier date and had soon found its way into many of the textbooks on algebra.[1] In fact, two separate books on determinants, viz., *Elementary Theorems Relating to Determinants,* 1851, by W. Spottiswoode, and *La Teorica dei Determinanti,* 1854, by F. Brioschi, had appeared before the date 1855, when the dictionary by Davies and Peck was "entered according to act of Congress."

The cited omission may serve to illustrate the fact that this dictionary was in many ways a half century behind the times when it was published, and hence it is simply ridiculous to claim that it has been "a standard work for 60 years." As evidence of the fact that in some respects it was actually more than a half century behind the times when it was published we need only cite the following definite statement, "The Calculus of Variations is the highest branch of mathematics," which appears under the term *mathematics* and also under *Calculus of Variations.* Another evidence of the same fact appears under the term *limit* in the following words: "A quantity towards which a varying quantity may approach to within less than any assignable quantity, but which it cannot pass."

The preceding remarks are not intended to convey the idea that the mathematical dictionary by Davies and Peck is of no value to the modern student of mathematics. They are, however, intended to convey the writer's conviction that as a work of reference for reliable information this dictionary cannot be recommended since it represents too many obsolete views and omits too many of the modern notions and developments. Some of these obsolete views are of much interest, partly because they represent approximately the stage of American mathematics a little more than half a century ago and also because they exhibit difficulties which the teacher is apt to overlook unless they are explicitly brought to his attention.

Among the obsolete mathematical methods which have received considerable attention in the history of elementary mathematics are those which relate to the four fundamental operations of arithmetic. Some of these methods are quite complicated from our present point of view but they often serve to give a deeper insight into the real meaning of these operations. This is true, in particular, as regards the operation of division.

Among the methods of division which were employed before the fifteenth century, when the present method began to be used, those included under the term complementary division were widely adopted during the Middle Ages.

[1] Cf. T. Muir, *The Theory of Determinants,* second edition, 1906, p. 14.

We proceed to give the principle of these methods by reproducing the explanations found in the *Encyclopédie des Sciences Mathématiques*, tome 1, volume 4, page 208.

Let A be the dividend, B the divisor having n figures. If

$$B = a_0 + a_1 \cdot 10 + a_2 \cdot 10^2 + \cdots + a_{n-1} \cdot 10^{n-1},$$
$$A = k_0 \cdot 10^n + k_1, \qquad \text{where} \qquad k_1 < 10^n,$$
$$d = 10^n - B,$$

it results that

$$\frac{A}{B} = \frac{k_0(B + d) + k_1}{B} = k_0 + \frac{k_0 d + k_1}{B}.$$

When $k_0 d + k_1 < B$ the desired quotient is k_0 and the remainder is $k_0 d + k_1$. When $B \leqq k_0 d + k_1 < 10^n$, the quotient is equal to k_0 increased by a number less than 10, which may be found by subtracting the divisor B a certain number of times from $k_0 d + k_1$. Finally, when $k_0 d + k_1 \geqq 10^n$ the original process may be repeated regarding $k_0 d + k_1$ as the dividend. Hence one can always find the quotient of A/B by a finite number of trials and by adding the partial quotients obtained in the manner just described.

We have reproduced here this explanation of an obsolete method partly because it may be of intrinsic interest to some of the readers of the MONTHLY and partly because it seems to serve well to illustrate the value of some of the obsolete in mathematics. Obsolete mathematical literature has its place but it should generally not be used until after one is familiar with the more recent literature. This principle should perhaps also be observed in mathematical history whose chief function should be to clarify modern developments and not to save the obsolete from oblivion.

As an instance of the usefulness of obsolete terms in mathematics we may refer to the "Rule of Three." If one reads about the arithmetical troubles of our grandparents one is apt to come across this term. Hence it is desirable to know that it is merely another name for proportion. Perhaps to future generations the terms variation and proportion will appear equally obsolete since these are merely names for special equations.

The Davies and Peck dictionary is rich in obsolete terms. While one cannot find the term *abelian* in this dictionary one does find two or three pages on *alligation, alligation medial, alligation alternate.* While one does not find the term *radian* one does find the term *sine of an arc,* defined as "the distance of an extremity of an arc upon the diameter drawn through the other extremity," and an explanation of the difference between sine of an arc and sine of an angle. We are also told that *to cipher* "is a common term, which is applied to the performance of any arithmetical operation by pupils."

Perhaps the most helpful lesson that the modern student of mathematics is apt to learn from such works as the mathematical dictionary by Davies and Peck is the great advantage of precise statements. While modern mathematical works are not always free from ambiguous statements one cannot help noticing a great improvement when one compares them as a whole with those written half a century ago. As an illustration we may cite the second sentence of the first article in the dictionary under consideration. "Among the ancients it

[the letter A] was used as a numeral denoting 500, or, with a dash over it, thus, $\bar{A}$, it stood for 500,000."

The question at once arises what ancients are meant. The number systems of the ancient Babylonians, the ancient Egyptians, the ancient Greeks, etc., do not reveal such a use of the letter A as a number symbol. From the fact that in medieval times the Romans sometimes used the horizontal bar above a number symbol to denote that the number represented by this symbol is to be multiplied by 1,000 leads one to suspect that the term "ancients" in this quotation probably refers to the Romans in the Middle Ages, but there is nothing in the article itself which would aid one in reaching this conclusion.

To teachers of mathematics the obsolete methods, viewpoints, and terms should be of peculiar interest since they involve elements which were once attractive, and were replaced by others which were still more attractive. The genesis of our modern methods and viewpoints may reveal possible further improvements, and a knowledge of the obsolete may enable us to assist more effectively in consigning to the obsolete those things which could now be replaced by the more useful.

A SUBSTITUTE FOR DUPIN'S INDICATRIX.

By C. L. E. MOORE, Massachusetts Institute of Technology.

1. **Dupin's Indicatrix.** Let the surface be given in the form $z = f(x, y)$ and let us take the origin at a non-singular point and take the tangent plane at the origin for the xy-plane. Then z can be expanded into an infinite series in x, y which will begin with the second-degree terms,

$$z = \tfrac{1}{2}(ax^2 + 2hxy + by^2) + \cdots, \tag{1}$$

where the terms omitted are of higher than the second order. One method of obtaining Dupin's indicatrix then is to write

$$ax^2 + 2hxy + by^2 = \pm 1. \tag{2}$$

If this conic is an ellipse then use the sign on the right, which will make it real and if it is an hyperbola use either sign. We note that in case this conic becomes a parabola it degenerates into two coincident lines and never takes the form of a general parabola. Points on the surface are then classified as elliptic, hyperbolic or parabolic according as the indicatrix is an ellipse, hyperbola or two coincident straight lines. The axes of the indicatrix correspond to the directions of the lines of curvature on the surface. The direction of the asymptotes of the indicatrix are the same as the direction of the asymptotic lines of the surface. Two directions on the surface are said to be conjugate if they coincide with conjugate diameters of the indicatrix. So we see that the indicatrix is quite intimately associated with the important directions on the surface.

It always seemed to me however that there was something arbitrary in the